"DOMAIN WALLS" IN MAGNETIC SUPERCONDUCTORS

by H. Suhl

ABSTRACT

A Landau Ginsburg theory is presented for the apparent first order transitions in the "magnetic superconductors" of the type $ErRh_4B_4$. The magnetic and superconducting order parameters are in competition because of both paramagnetic and diamagnetic coupling. I discuss the extreme cases in which one or the other coupling dominates.

I. INTRODUCTION

One of the many problems in physics clarified by Felix Bloch is that of the spatial variation of magnetization in ferromagnets. In the materials available at the time, he advanced the notion of what is now known as the Bloch wall, in which the only spatial changes that could be expected were changes in direction, not in magnitude, of the magnetization. A relatively slow turning of the magnetization, extending over a hunded angstroms or so, does not require much energy, about as much as the electromagnetic energy residing in fringing fields. Thus the introduction of these walls will sometimes lower the total energy because, suitably disposed, they inexpensively lower the magnetic energy stored in the fringing field. Spatial variations in the *magnitude* of **M** were not needed.

Very recently a new class of materials has been found[1] that seems to involve another kind of domain wall: walls across which the *magnitude* changes in space. In these materials magnetic ordering is in competition with the order characterizing superconductivity. They are metallic compounds of magnetic rare earth ions in a superconducting matrix. A proto-

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type is $ErRh_4B_4$. (For a detailed description, particularly of the systematic variation of the magnetic-superconducting properties with substitution of other rare earths, see references 2 and 3.) The effective coupling responsible for the magnetism is so weak that the magnetic order sets in at a temperature T_c well below the superconducting transition temperature T_s (in fact for $ErRh_4B_4$, we have $T_s = 8.7\text{K}$, $T_c \simeq 1\text{K}$). However, once the magnetic order sets in, it tends to diminish or destroy the superconductivity. The reason is that superconductivity (as far as we know to date) requires antiparallel pairing of conduction electron spins. But these spins are in exchange interaction with the magnetic ions (Er in the present case). If they occur only in antiparallel pairs, they cannot acquire a net spin polarization from the aligned magnetic ions, and must sacrifice the polarization energy they would have were they not bound in pairs. When that energy exceeds the pairing energy, the superconductivity tends to be destroyed. It is possible that in some situations a compromise is reached. For example, if the magnetization, instead of aligning uniformly, arranges itself in a spiral order, with several turns of the spiral within the range of the electron-pair wavefunction, the pair can ignore the disruptive effect of the exchange field.[4,5] This possibility aside, there are only two other obvious ways in which coexistence of both phases (in the same volume element) could occur: 1) p-wave superconductivity, in which the pair-wavefunction is antisymmetric and therefore requires parallel-spin pairing, and 2) a ferromagnetic order not mediated by the conduction electrons (the Ruderman-Kittel-Kasuya-Yosida interaction). As for 1), there is so far no experimental evidence for p-wave pairing. As for 2), one could envisage dipolar ferromagnetism (in suitable non-cubic structures), or one could perhaps invoke RKKY coupling by a class of electrons not involved in the superconductivity.

If the possibility 2) occurred in $ErRh_4B_4$, it would be difficult to explain why the superconductivity would cease below T_c, since the relevant electrons no longer see the magnetism. The only pair-breaking effect they would see would be due to coupling of their orbits to any magnetic fields from the aligned ions. Although I doubt that this can be the dominant mechanism, the possibility is analyzed toward the end of this paper. I begin by analyzing the more likely situation: that of significant exchange coupling of conduction electrons to Erbium spins, and for the purposes of this paper I treat the resulting RKKY interaction as the only source of magnetic ordering. Figure 1 shows a schematic plot of the magnetic free energy F_m when the superconductivity is ignored, and the superconducting free energy F_s when the magnetism is ignored, for the case $T_c^0 < T_s^0$ and $F_m(T=0) < F_s(T=0)$. If there were no coupling, the two phases would coexist and the total free energy would be $F_s + F_m$. When a coupling of

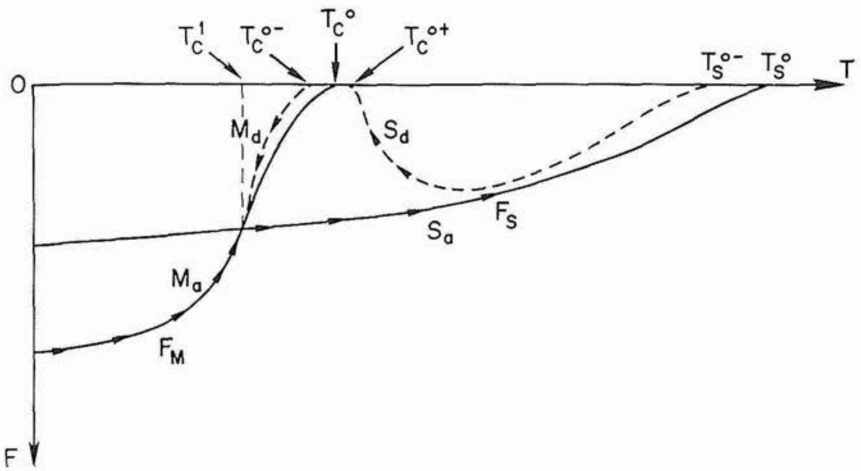

FIG. 1. FREE ENERGY VERSUS ASCENDING AND DESCENDING TEMPERATURE, disregarding the possibility of spiral magnetic structure. In the ascending case a first order transition should occur at T_c^1. In the descending case, the transition should be second order near T_c^0. (S—superconductivity, M—magnetic, a—ascending, d—descending.)

sufficient strength is turned on, we see from the figure that an interesting hysteretic situation should arise:

1. Suppose the temperature is lowered from $T > T_s^0$. The sequence of events is as follows:
   a) At a temperature T_s^{0-} somewhat below T_s^0, the sample goes superconducting. T_s is diminished from T_s^0 by the electron coupling to the paramagnetic fluctuations of the magnetization.[6]
   b) Slightly above T_c^0, at T_c^{0+}, the spin fluctuations become critically large, and diminish the superconducting order parameter continuously to zero. Slightly below T_c^0, at T_c^{0-}, a small net magnetization begins to build up from zero. Thus we have a second order transition. (In the very narrow range $T_c^{0-} < T < T_c^{0+}$ the situation is unclear and requires a full renormalization treatment of the transition.)
   c) From T_c^{0-} downwards, there is no more superconductivity (only, perhaps, a fluctuation enhancement of the normal conductivity).
2. Suppose the temperature is raised from $T = 0$.
   a) The sample stays magnetic up to the cross-over at T_c^1 (figure 1). At this point a first order transition should occur. T_c^1 is near neither "natural" transition temperature T_c^0 nor T_s^0. Therefore there are no critical fluctuations in either order parameter capable of reducing the other continuously to zero.

b) As with any first order transition, superheating effects become possible, provided the surface energy between superconducting and magnetic regions is positive.

c) If the superheating can be maintained up to $\sim T_c^{0-}$, a second order transition to the non-magnetic, superconducting state occurs.

This picture is further complicated if so called crypto-magnetic states are admitted. Then as T_c^{0+} is approached from above, spiral ordering could set in without destruction of superconductivity. This transition is weakly first order (the superconducting order parameter undergoes a small discontinuous reduction). Detailed calculations have not yet been performed concerning the further progress as T is diminished. Quite possibly a first order transition to the fully aligned magnetic state eventually occurs.

II. THE FREE ENERGY DENSITY

In this paper I discuss the domain wall energy between magnetic and superconducting regions at temperatures near T_c^1, using the Ginsburg Landau equations for the coupled magnetic and superconducting order parameters M and ψ respectively. Strictly speaking, this is valid only near transitions at which both M and ψ are small, which is not necessarily the case near T_c^1, but the results of such a theory usually are qualitatively valid beyond this limit. We take the Ginsburg-Landau free energy density in the form

$$F = F_S + F_M + F_{SM}$$

where

$$F_S = \frac{\hbar^2}{2m}|\nabla\psi|^2 - \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4$$

$$F_M = \frac{1}{2}d^2(\nabla M)^2 - aM^2 + \frac{1}{2}bM^4$$

$$F_{SC} = \frac{1}{2}pM^2|\psi|^2 \quad . \tag{1}$$

Here p is a constant that is positive to ensure that the tendencies to the two types of ordering are in conflict. b and β are positive and almost constant. Also, $a = a_o(T_c^0 - T)$, $\alpha = \alpha_o(T_s^0 - T)$ where a_o and α_o are constants. The use of $\hbar^2/2m$ in F_s is a mere convention; the relevant physical parameter is the superconducting coherence length ξ given by

$$\xi^2 = \hbar^2/(2m\alpha) \quad .$$

d is related to the magnetic coherence length brought about by exchange stiffness. In terms of a and d the magnetic coherence length is l, where

$$l^2 = d^2/a \quad .$$

The energies F_s and F_m separately have minima at $|\psi|^2 = \alpha/\beta$ and at $M^2 = a/b$, both independent of position, and the corresponding energies are $-\alpha^2/2\beta$ and $-a^2/2b$ respectively. In the coupled situation it is therefore useful to introduce reduced variables f and μ according to

$$\psi = \sqrt{\frac{\alpha}{\beta}}\, f$$

$$M = \sqrt{\frac{a}{b}}\, \mu$$

so that the total free energy becomes

$$F = \frac{\alpha^2}{\beta}\left(\xi^2(\nabla f)^2 - f^2 + \frac{1}{2}f^4\right) +$$

$$\frac{a^2}{b}\left(l^2(\nabla\mu)^2 - \mu^2 + \frac{1}{2}\mu^4\right) + \frac{1}{2}\frac{pa\alpha}{b\beta}\mu^2 f^2 \quad .$$

The forms of f and μ that make $\int F dv$ stationary satisfy

$$\xi^2\nabla^2 f + f[1 - (f^2 + \gamma_1\mu^2)] = 0$$

$$l^2\nabla^2\mu + \mu[1 - (\mu^2 + \gamma_2 f^2)] = 0 \qquad (2)$$

where $\gamma_1 = pa/2\alpha b$ and $\gamma_2 = pa/2a\beta$. Now at T_c^1, which is defined by $(F_S)_{\min} = (F_M)_{\min}$, we evidently have $\alpha^2/\beta = a^2/b$, so that at T_c^1, γ_1 and γ_2 become equal, and their common value is

$$\gamma = p/(2\sqrt{\beta b}) \quad .$$

Since we are interested in the domain wall energy only in the vicinity of T_c^1, we shall assume this common value.

Also, we wish specifically to exclude the possibility of coexistence of uniform nonzero values of both μ and f. The uniform solutions of

$$\xi^2\nabla^2 f + f[1 - (f^2 + \gamma\mu^2)] = 0$$

$$l^2\nabla^2\mu + \mu[1 - (\mu^2 + \gamma f^2)] = 0 \tag{3}$$

are the usual ones ($f=0$, $\mu=1$) and ($f=1$, $\mu=0$) and in addition, the "coexistence" solutions

$$f^2 + \gamma\mu^2 = 1$$

$$\mu^2 + \gamma f^2 = 1$$

which give $\mu^2 = f^2 = 1/(1+\gamma)$. We require that this last one be a maximum or a saddle point of F rather than a minimum.

Writing $F = \frac{a^2}{2b}\tilde{F}$, it turns out that at that point $\tilde{F}^2_{\mu f} - \tilde{F}_{\mu\mu}\tilde{F}_{ff} = 16\frac{\gamma-1}{\gamma+1}$, so that it is indeed a saddle point for $\gamma > 1$, whereas ($f=0$, $\mu=1$) and ($f=1$, $\mu=0$) are both minima. When $\gamma < 1$, since $\frac{\partial^2\tilde{F}}{\partial f^2} = -2 + 6f^2 + 2\gamma\mu^2 = \frac{4}{1+\gamma}$ at $f^2 = \mu^2 = \frac{1}{1+\gamma}$, that point is a minimum, and the energy there is $\alpha - f^2 + \frac{1}{2}f^4 - \mu^2 + \frac{1}{2}\mu^2 + \gamma\mu^2 f^2 = -\frac{1}{1+\gamma}$. The points ($f=0$, $\mu=1$) and ($f=1$, $\mu=0$) are saddle points when $\gamma < 1$.

We shall here be concerned with the case $\gamma > 1$. The two minima ($f=0$, $\mu=1$) and ($f=1$, $\mu=0$) with energies $\tilde{F}_{\min} = -\frac{1}{2}$ are then separated by a barrier energy $-\frac{1}{1+\gamma}$, which is the energy of the saddle point $f^2 = \mu^2 = \frac{1}{1+\gamma}$. Unless it turns out that spatial gradients can lower the energy, there will be a barrier to the growth of one kind of domain at the expense of the other as T_c^1 is traversed. This surface energy can be found by solving Eqs. (3).

[We note here that the form $\sim m^2|\psi|^2$ for the coupling energy is not always adequate. As shown elsewhere,[4] for a discussion of the second order transition as T decreases through T_c^0, it is necessary to take into account the non-local character of ψ. If this is not done and the form $m^2|\psi|^2$ is adopted, the effect of the magnetic fluctuations on ψ becomes temperature independent, as the result of a sum-rule. In the discussion of a first order transition, however, taking this matter into account would only produce a refinement of the conclusions.] To find the surface energy we need a solution of the one-dimensional case of Eq. (3), for $\gamma > 1$, in which f varies from 1 to 0 and μ from 0 to 1 as x varies from $-\infty$ to $+\infty$. We discuss three cases:

a) $l << \xi$
b) $\xi << l$
c) $\gamma >> 1$

Case a). μ adjusts itself to the local value of f, because $l\nabla\mu^2$ is negligible:

$$\mu^2 = 1 - \gamma f^2$$

in the region in which $f^2 < \dfrac{1}{\gamma}$. Where $f^2 > \dfrac{1}{\gamma}$ we have $\mu = 0$. Thus we must solve

$$\xi^2 f'' + (1 - \gamma) f[1 - f^2(1 + \gamma)] = 0 \quad , \quad f^2 < \frac{1}{\gamma} \tag{4}$$

and

$$\xi^2 f'' + f(1 - f^2) = 0 \quad , \quad f^2 \geq \frac{1}{\gamma} \tag{5}$$

subject to $f = 1$, at $x = -\infty$ and $f = 0$ at $x = +\infty$.

For $f^2 \geq \dfrac{1}{\gamma}$ the solution has the form

$$f = \frac{1 - A\, e^{2\sqrt{2}x/\xi}}{1 + A\, e^{2\sqrt{2}x/\xi}}$$

where A is a constant.

For $f^2 < \dfrac{1}{\gamma}$, we write $f\sqrt{1+\gamma} = \varphi$, $\dfrac{\xi^2}{\gamma - 1} = \bar{\xi}^2$ in which case Eq. (4) becomes

$$\bar{\xi}^2 \varphi'' - \varphi(1 - \varphi^2) = 0 \quad .$$

The solution subject to $f = 0$ at $x = +\infty$ is

$$f = \frac{2\sqrt{2}Be^{x/\xi}}{(1 + B^2e^{2x/\xi})\sqrt{(\gamma+1)}} \quad .$$

A and B are found from the requirement that these expressions be equal to $\frac{1}{\sqrt{\gamma}}$ at a particular x, say $x=0$. This gives $A=\frac{\sqrt{\gamma-1}}{\sqrt{\gamma+1}}$, and $B=(\sqrt{2\gamma}\pm\sqrt{\gamma-1})/\sqrt{\gamma+1}$. The choice of sign is made as follows: Because of the approximation ($\mu^2=1-\gamma f^2, f^2<\frac{1}{\gamma}$), $\mu^2=0$ elsewhere, a discontinuity of slope arises at $x=0$. The slope of the exact solution of course has no such discontinuity. We chose the sign in the expression for B such that the discontinuity is least obtrusive. For $f^2\geq\frac{1}{\gamma}$ the slope is negative, equal to $\frac{-\sqrt{2}}{2\gamma\xi}$ $(\gamma-1)$, whereas for $f^2\leq\frac{1}{\gamma}$ it is $\mp\frac{\sqrt{2}\sqrt{\gamma-1}}{2\xi\gamma(\gamma+1)}$. It follows that the upper case should be chosen

$$B = (\sqrt{2\gamma} + \sqrt{\gamma - 1})/\sqrt{\gamma + 1}$$

in order that the slope, though discontinuous, be at least negative on both sides of $x=0$.

Case b). By symmetry, the solution can be derived from the solution of case a) by the replacements: $f\leftrightarrow\mu$ and $\xi\leftrightarrow l$, $\bar{\xi}\leftrightarrow\bar{l}$ where $\bar{l}=l/\sqrt{\gamma-1}$.

Case c). This case is the simplest (and quite possibly the most relevant). When $\gamma>>1$ we evidently have $f(x)=0$ $x\geq0$, and $\mu(x)=0$ $x\leq0$. Thus we solve

$$\xi^2f'' + f(1 - f^2) = 0$$

subject to $f=1, x=-\infty$, and $f=0, x=0$. A first integral is

$$\frac{\xi^2}{2}(f')^2 + \frac{1}{2}f^2 - \frac{1}{4}f^4 = \text{const.}$$

and, since $f'=0$ at $x=-\infty$, the constant is $+\frac{1}{4}$. Further integration, subject to $f(-\infty)=1, f(0)=0$ gives

$$f(x) = \frac{1 - e^{\sqrt{2}x/\xi}}{1 + e^{\sqrt{2}x/\xi}} \quad , \qquad x < 0$$

$$= 0 \qquad\qquad , \qquad x < 0 \quad .$$

In exactly the same way

$$\mu(x) = \frac{1 - e^{-\sqrt{2}x/\xi}}{1 + e^{-\sqrt{2}x/\xi}}\,, \qquad x > 0$$

$$= 0 \qquad\qquad , \qquad x < 0 \quad .$$

The free energy associated with this configuration is (recall that $a^2/2b = \alpha^2/2\beta$ at T_c^1),

$$\frac{a^2}{b} \int \left\{ \left[\frac{\xi^2}{2} (f')^2 - \frac{1}{2} f^2 + \frac{1}{4} f^4 \right] \theta(-x) + \left(\frac{l^2}{2} \mu'^2 - \frac{1}{2} \mu^2 + \frac{1}{4} \mu^4 \right) \theta(x) \right\} dx$$

$$= \frac{a^2}{b} \int \left\{ \left[\xi^2 (f')^2 + l^2 (\mu')^2 \right] - \frac{1}{4} \right\} dx$$

where $\theta(x) = 0$ or 1 according as $x \lessgtr 0$.

On the other hand, the energy of either the uniformly magnetic or uniformly superconducting state is $-a^2/4b \int dx$. Thus it follows that the surface energy is positive, and equal to

$$E_S = \frac{a^2}{b} \left[\xi^2 \int_{-\infty}^{0} (f')^2 dx + l^2 \int_{0}^{\infty} (\mu')^2 dx \right]$$

$$= \frac{a^2}{\sqrt{2} b} (l + \xi) \int_0^\infty \frac{dx}{\cosh^4 x}$$

$$= \frac{2}{3\sqrt{2}} \frac{a^2}{b} (l + \xi) \quad .$$

From this we can estimate the temperature width of the hysteretic region. Suppose one is going up in temperature from $T = T_c^1$ to $T = T_c^1 + \Delta T$. The cost of maintaining a volume $\frac{4}{3}\pi r^3$ of the sample in the magnetic state is

$$-\frac{4}{3}\pi r^3\left[\frac{a_0^2\,(T_c^1+\Delta T-T_c^0)^2}{2b}-\frac{\alpha_0^2\,(T_c^1-T_s)^2}{2\beta}\right]$$
$$=\frac{4}{3}\pi r^3\left[\frac{a_0^2\,(T_c^0-T_c^1)\Delta T}{b}\right]\quad .$$

On the other hand the surface energy is $4\pi r^2 E_S$. The two are equal when

$$\frac{r}{3}\Delta T = (l+\xi)(T_c^0-T_c^1)\quad .$$

But superconducting nuclei cannot possibly be smaller than $r \sim \xi$; therefore, the maximum hysteresis width on the way up is

$$\Delta T = 3\left(1+\frac{l}{\xi}\right)(T_c^0-T_c^1)\quad .$$

Similarly on the way down it is

$$\Delta T' = 3\left(1+\frac{\xi}{l}\right)(T_c^0-T_c^1)\quad .$$

III. DIAMAGNETIC EFFECTS

Maekawa and Tachiki[7] have made a detailed study of the diamagnetic aspects of these materials when they are in the paramagnetic regime, i.e., above T_c^0. It is of some interest to consider these effects at and below T_c^0 also, even though most likely the exchange coupling of the superconducting electrons to the localized spins dominates the character of the transition. We consider here only an extreme situation, one in which the ordering mechanism of the rare earth spins does *not at all* involve the superconducting electrons. For example, for low enough symmetry of the magnetic sites, the ferromagnetism might be of dipolar origin, or conceivably, it may arise by indirect interaction via conduction electrons belonging to a band "orthogonal" to that of the superconducting electrons. Here we consider only this latter case.

The total free energy now takes the form

$$F = F_S + F_M + H^2/8\pi - \mathbf{H}\cdot\mathbf{M}$$

where F_M has the same form as before and

$$F = \frac{1}{2m} \left| \left(-i\hbar\nabla - \frac{2e\mathbf{A}}{c}\right)\psi\right|^2 - \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \quad .$$

F_{SM}, by hypothesis, is absent, and the coupling of the two order parameters now takes place through the electromagnetic field only.

The resulting variational equations are

$$\frac{1}{2m}\left(-i\hbar\nabla - \frac{2e\mathbf{A}}{c}\right)^2\psi - \alpha\psi + \beta|\psi|^2\psi = 0 \tag{6a}$$

$$-d^2\nabla^2\mathbf{M} - a\mathbf{M} + bM^2\,\mathbf{M} - \mathbf{H} = 0 \tag{6b}$$

$$\begin{aligned}\nabla\times\nabla\times\mathbf{A} &= -\frac{4\pi e\hbar}{imc}(\psi^*\nabla\psi - \psi\nabla\psi^*)\\ &\quad - \frac{16\pi e^2}{mc^2}|\psi|^2\mathbf{A} + 4\pi\nabla\times\mathbf{M} \quad .\end{aligned} \tag{6c}$$

For the one-dimensional variation considered here, with $\mathbf{M} = (0, 0, M(x))$, $\mathbf{A} = (0, A(x), 0)$, $\mathbf{H} = (0, 0, H(x))$, $\psi = fe^{i\theta}$, $\theta =$ constant, these equations reduce to

$$\begin{aligned} -\frac{h^2}{2m}\frac{\partial^2 f}{\partial x^2} - \alpha f + \beta f^3 + \frac{2e^2}{mc^2}A^2 f &= 0 \\ -d^2\frac{\partial^2 M}{\partial x^2} - aM + bM^3 - H &= 0 \\ -\frac{\partial^2 A}{\partial x^2} + \frac{A}{\lambda^2} + 4\pi\frac{\partial M}{\partial x} &= 0 \\ \frac{\partial H}{\partial x} &= \frac{A}{\lambda^2} \end{aligned} \tag{7}$$

$$\lambda^2 = mc^2\beta/(16\pi e^2\alpha) \quad .$$

Note that Eqs. (6) at once give the results of Maekawa and Tachiki for the paramagnetic case $a = -|a|$, at least for the case of very small d. Then $\mathbf{M} = \mathbf{H}/|a| = \chi\mathbf{H}$ where χ is the paramagnetic susceptibility. Since $\nabla \times \mathbf{H} = 4\pi\mathbf{J}/c$, the last equation of (6) then reads $\nabla \times \nabla \times \mathbf{A} = \frac{4\pi}{c}\mathbf{j}(1 + 4\pi\chi)$. For the case of a constant phase of ψ, this may be written $\nabla \times \nabla \times \mathbf{A} = -\mathbf{A}/\lambda^2$, where

$$\lambda_{\mathrm{eff}}^2 = \lambda^2/(1 + 4\pi\chi)$$

so that, as T_c^0 is approached from above, and χ becomes large, the effective penetration length shortens, and the superconductor tends to have type I properties. Sufficiently below T_c^0, the same conclusion applies. Sufficiently below T_c^0, it is possible to write $M = M_0 + \chi_F H$, with $M_0 = \mathrm{const}$. For very small l, the second of Eqs. (6) gives

$$\chi_F\,(3bM_0^2 - a) = 1$$

where

$$\chi_F = 1/2a$$

and now $\lambda_{\mathrm{eff}}^2 = A^3/(1 + \chi_F)$. Thus below T_c^0, also, there is some tendency towards type I behavior. Very close to T_c^0, the variation of M with H is non-analytic. To obtain some idea of the behavior there, we consider the case $T = T_c^0$, neglecting as before the $d^2\nabla^2\mathbf{M}$ term.

Then $a = 0$, and $M = (H/b)^{1/3}$ and so we have

$$-\frac{\partial^2 A}{\partial x^2} = -A/\lambda^2 - 4\pi\frac{\partial}{\partial x}\left(\frac{H}{b}\right)^{1/3}$$

together with

$$\frac{\partial H}{\partial x} = A/\lambda^2 \quad .$$

It follows that

$$\frac{\partial^2 H}{\partial x^2} = (H + 4\pi(H/b)^{1/3})/\lambda^2 \tag{8}$$

if we suppose that $H''=0$ when $H=0$. A first integral of (8) is

$$\left(\frac{\partial H}{\partial x}\right)^2 = (H^2 + (6\pi/b)^{1/3} H^{4/3})/\lambda^2$$

if $H'=0$ when $H=0$. We seek a solution that has $H=0$ at $x=\infty$ and $H=H_0$, the applied tangential field at the sample boundary. The substitution $H=y^3$ turns this equation into

$$\int \frac{dy}{\sqrt{y^2 + \left(\frac{6\pi}{b}\right)^{1/3}}} = (-x + \text{const.})/3\lambda$$

(the minus sign being chosen to obtain decay in the $+x$ direction). The results are $y=(6\pi/b)^{1/3}$ [$\sinh(\Lambda-x)/3\lambda$], whence $H=(6\pi/b)\sinh^3[(\Lambda-x)/3\lambda]$. The constant Λ is found from $H_0=(6\pi/b)\sinh^3(\Lambda/3\lambda)$. This solution persists up to $x=\Lambda$. For $x>\Lambda$, the solution must be $H=0$. The derivatives $(\partial^n H/\partial x^n)_{x=\Lambda+0}$ and $(\partial^n H/\partial x^n)_{x=\Lambda-0}$ are continuous up to and including $n=2$, so that no difficulties arise with the solution (8). Hence at $T=T_c^0$ we have a field-dependent penetration depth, equal to

$$\Lambda = 3\lambda \sinh^{-1}\left(\frac{bH_0}{6\pi}\right)^{1/3} \quad ,$$

which tends to zero with H_0. This suggests that at $T=T_c^0$, the behavior is type I. Thus there will always be a certain neighborhood around T_c^0 in which type I behavior may be expected. Accordingly, the surface energy will be positive, and the transition may be expected to be first order and hysteretic.

IV. STATES WITH SPIRAL MAGNETIZATION

In a previous publication, I discussed states with uniform ψ and spiral M that were solutions of the Ginsburg Landau equations when only the exchange mechanism is active. In this section we note that such a solution also exists when only the indirect diamagnetic effects of the magnetization are considered.[8] To see this, we write $\psi=\text{const.}=f$, $\mathbf{M}=(M_0\cos qz, M_0 \sin qz, 0)$. $\mathbf{H}=(h\cos qz, h\sin qz, 0)$, $\mathbf{A}=-\lambda^2\nabla\times\mathbf{H}=-\lambda^2 hq(\cos qz, \sin qz, 0)=-\lambda^2 q\mathbf{H}$ in Eqs. (6), with the result

$$\alpha = \beta f^2 + \frac{\lambda^2}{8f^2} q^2 h^2 \qquad \text{(from 6a)} \qquad (9a)$$

$$bM_0^2 = a - d^2q^2 + \frac{h}{M_0} \qquad \text{(from 6b)} \tag{9b}$$

$$h = -4\pi M_0/(1 + q^2\lambda) \qquad \text{(from 6c)} \quad . \tag{9c}$$

The usual solution is $q=0$ and gives $f^2=\alpha/\beta$, $M_0^2=(a-4\pi/b)$ and $h=-4\pi M_0$ corresponding to complete expulsion of the flux B. Note that the effective T_c^0 is depressed as the result of the expulsion: $k_BT_c^0 \to k_BT_c^0 - 4\pi/a_o$; in whichever direction the magnetization attempts to form, the field due to the flux expulsion attempts to reduce it. For finite q this effect will be diminished provided

$$d^2q^2 + \frac{4\pi}{1+q^2\lambda^2} < 4\pi$$

i.e., provided

$$q^2\lambda^2 < \frac{4\pi\lambda^2}{d^2} - 1 \quad .$$

This can always be arranged provided $\lambda > d/2\sqrt{\pi}$, which is normally satisfied. The flux expulsion is no longer total, a condition that favors the establishment of the full magnetization. To see what a state with $q \neq 0$ implies for the gap parameter f, we note that

$$\lambda^2 = \frac{mc^2}{16\pi e^2 f^2} \quad , \quad m = \frac{2eh\varkappa}{c}\left(\frac{2\pi}{\beta}\right)^{1/2}$$

where $\varkappa = \lambda/\xi$. Then the equation for f becomes (with ϕ_0 the quantum of flux),

$$\alpha = \beta f^2 + \frac{\phi_0\varkappa}{4\pi}\left(\frac{2\pi}{\beta}\right)^{1/2} \frac{q^2M_0^2}{\left(f^2 + \frac{\phi_0\varkappa q^2}{\lambda\pi^2}\left(\frac{2\pi}{\beta}\right)^{1/2}\right)^2}$$

This equation has a solution (evolving continuously from $f^2=\alpha/\beta$ as q^2 is increased from zero) provided that

$$\alpha > 2^{-2/3}\left(\frac{\phi_0\varkappa}{4\pi}\left(\frac{2\pi}{\beta}\right)^{1/2} q^2 M_0^2\right)^{1/3}\beta^{2/3} - \frac{\phi_0\varkappa q^2}{8\pi^2}\left(\frac{2\pi}{\beta}\right)^{1/2} .$$

(Strictly speaking, the effect on M_0 of the change in f according to Eq. (9b) and (9c) should also be considered, but is presumably small and is neglected here.) The value of f^2 is largest at $q=0$; therefore a state with $q\neq 0$ involves some sacrifice of superconducting energy. This loss may well be more than compensated by a lowering of magnetic energy for $q\neq 0$. However, at the present stage of experimental work it seems premature to follow this speculative line any further.

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